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PRODUCING DESIRED COLOR STIMULI WITH TRICHROMATORS: A TUTORIAL ON TECHNIQUE

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VISUAL DISPLAY SYSTEMS BRANCH
HUMAN ENGINEERING DIVISION

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NOTICES

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This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

CHARLES BATES, JR.
Director, Human Engineering Division
Air Force Aerospace Medical Research Laboratory

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A summary of the theory of color mixing when using a trichromator system is presented. The derivation of needed formulae to compute any desired chromaticity coordinates and corresponding luminance requirements is given. A numerical example is provided which demonstrates the application of the formulae.

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PREFACE

The theory of color mixing described herein was developed and used during a cooperative research program between the Air Force Aerospace Medical Research Laboratory, Human Engineering Division, Visual Display Systems Branch and the Human Factors Group, Royal Aircraft Establishment, Farnborough, England.

The author is a member of the Visual Display Systems Branch and was assigned to the Royal Aircraft Establishment under NATO Working Party 61 of the Aircrew Standardization and Coordinating Committee (ASCC) from December 1982 - July 1984. The research effort was funded by the European Office of Aerospace Research and Development under AFOSR Grant 83-0085. The author's participation was under work unit 71841144.

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1. SUMMARY

This report summarizes the theory of color mixing when using a trichromator system. The four-channel colorimeter system described is of a generic nature. The derivation of formulae needed to compute chromaticity coordinates and luminance requirements are presented. A numerical example is provided which demonstrates the application of the formulae.

2. INTRODUCTION

The need sometimes arises in laboratory experimentation to produce stimuli of precise chromaticity and luminance. The need this author has encountered involves color matching experiments, where two stimuli are generated that must be matched along one dimension (either dominant wavelength, excitation purity, or luminance). In the author's case, the two stimuli formed a bipartite field (Greene & Laycock, 1984). One-half of the matching field was set by the experimenter to be a predetermined and fixed chromaticity and luminance. The subject adjusted the other half of the bipartite field for a match in one of the above mentioned dimensions.

To produce the two stimuli for color-matching, a trichromator system is described which mixes the appropriate wavelengths in proper luminance proportions to generate the bipartite field. Although this report describes a generic colorimeter, it is representative of ones currently existing in laboratories in the Crew Systems Effectiveness Branch, Air Force Aerospace Medical Research Laboratory (AFAMRL/HEF), Wright-Patterson Air Force Base, Ohio, and in the Human Factors Group, Royal Aircraft Establishment, Farnborough, England.

3. TRICHROMATOR DESCRIPTION

To generate stimuli for color-matching experiments, a trichromator system was chosen by this author to produce light sources of different wavelengths for mixing. Depending upon how much luminance is required of the stimuli, the light source for the trichromator system can either be a single source divided among the three monochromators, or multiple light sources, one for each of the three monochromators.

Because of the exact nature of the mathematical equations used in the calculation of wavelengths, it is desirable to have monochromators whose selectable wavelengths are accurate to 0.1 nm. Otherwise, the mixing of wavelengths will result in less than desired precision in resultant chromaticity coordinates.

With a trichromator, monochromatic light from the three monochromators is combined through four optical channels in such a way as to produce two stimuli. In order to do this, one of the monochromators is fixed at an anchor wavelength, usually in the longer wavelengths of the visible spectrum (i.e., 600+ nm). The light from the anchor wavelength is split in half by use of a beam-splitter, forming two channels. Light from one of the other two monochromators is combined with one channel of the anchor wavelength, in exact quantities of luminance from each, to produce one stimulus of a precise chromaticity. In the same way, the third monochromator combines monochromatic light of a specific luminance with the other channel of the anchor wavelength to produce a second stimulus, again of a required chromaticity.

Figure 1 shows diagrammatically the trichromator system presently configured at the Royal Aircraft Establishment. The four optical channels are labelled for ease of understanding. In this example, light from channels 2 and 4 form the anchor wavelength channels. The monochromator in channels 2 and 4 is always set at the anchor wavelength. The light output from channels 1 and 2 is mixed to form the chromaticity, of a controlled luminance, required for one of the stimuli. Similarly, the output from channels 3 and 4 is combined to produce the chromaticity of the second stimulus, again at a precise luminance level. This combination of light outputs is shown in Figure 1. The AFAMRL trichromator system is nearly identical.

4. COLOR MIXING

The tristimulus values resulting from the mixture of two or more lights are the sums of the lights' individual tristimulus values. For a mixture consisting of only two lights, the point representing the chromaticity of the mixture is located on the straight line connecting the points representing the chromaticities of the two lights on one of the Commission Internationale de l'Eclairage (CIE) chromaticity diagrams (Graham, 1965, pp. 270-294). Any of the CIE chromaticity diagrams may be used for the purposes described herein, but the present discussion will deal only with the 1931 CIE diagram, as it is the most familiar. It is this fact of color mixing which describes the basis for the theory of the trichromator system. On the straight line drawn between two points on the spectrum locus of the 1931 CIE diagram lie all the chromaticity coordinates achievable by the mixture of these two lights.

There are three steps this author went through to use the trichromator system for color mixing. The first was to select the anchor wavelength for mixing. It was the author's desire to produce stimuli at a fairly high luminance level - 2000 cd/m^2. It was this consideration, coupled with the desire to examine dominant wavelengths in the longer end of the visible spectrum (640 nm), which influenced the choice of the anchor wavelength. The luminance calibration of the light output from the monochromator in channels 2/4 revealed the need for an anchor wavelength of 650 nm for the particular color-matching experiment of interest.

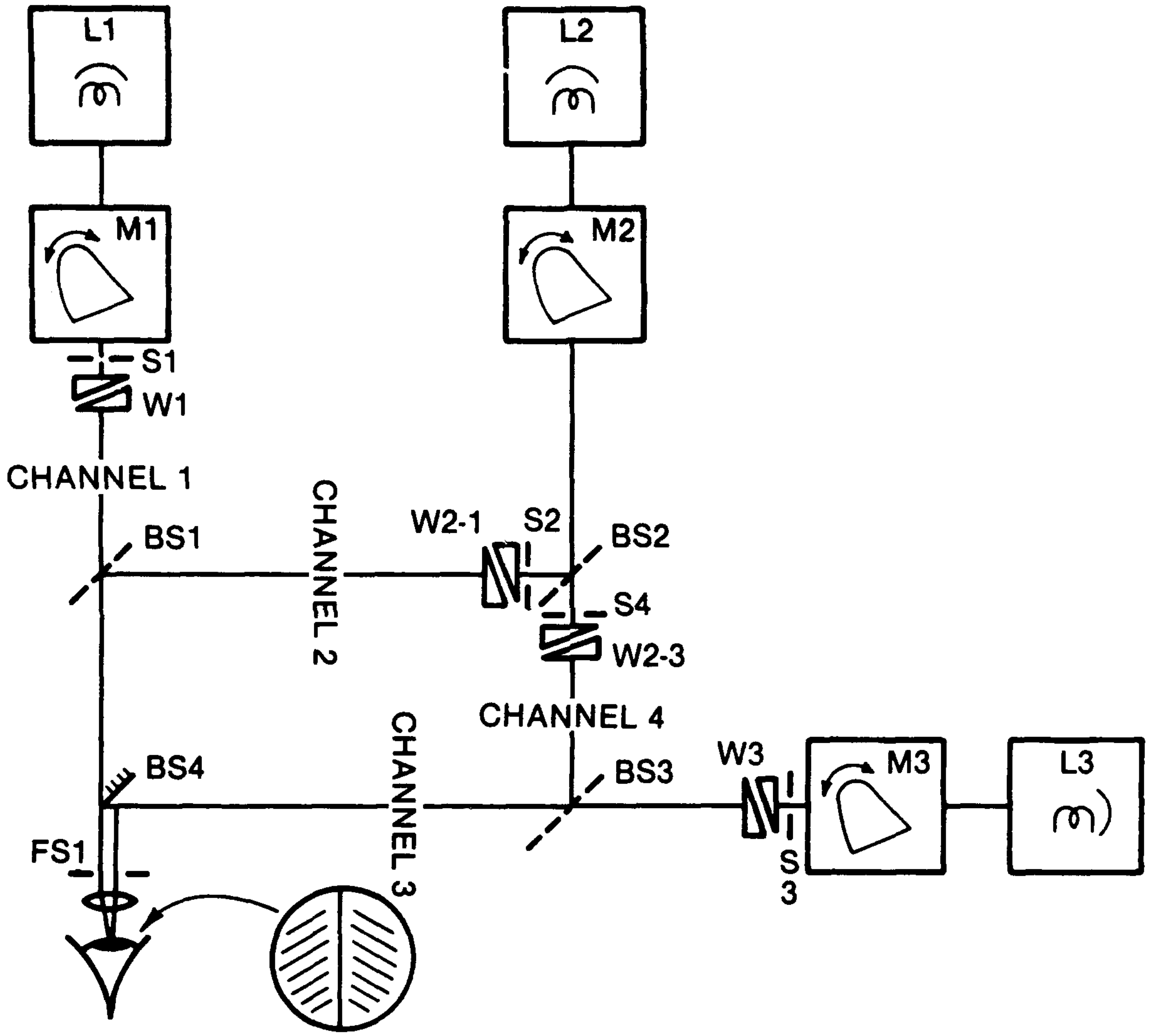

Figure 1. Optical Representation of the Trichrimator Hardware.

The second step was to calculate the variable wavelength for mixture with the anchor wavelength. This variable wavelength determination is dependent upon what stimulus chromaticity required.

The third step was to calculate the luminance proportions of the anchor and variable wavelengths required to produce a given stimulus chromaticity at a precise luminance. The next two sections describe the calculations required to achieve steps 2 and 3. The anchor wavelength setting can vary depending upon the experimental conditions under investigation.

4.1 Computation of Variable Wavelength

Figure 2 illustrates the mixing of wavelengths within a chromaticity diagram. Point "A" on the diagram shows the location on the spectrum locus of the chosen anchor wavelength, 650 nm in this example. Point "B" represents the desired chromaticity within the diagram. In order to determine which wavelength must be combined with the anchor wavelength to produce the desired chromaticity, the following procedure is used. A straight line is drawn from the anchor wavelength on the spectrum locus (Point "A") through the desired point within the chromaticity diagram (Point "B"), and extended to where the line intersects the spectrum locus, 493 nm in this example (Point "C"). The dominant wavelength represented by Point "C" is what is required for mixing with the anchor wavelength to produce the desired chromaticity.

It is possible to produce any desired chromaticity within the diagram by the combination of two dominant wavelength sources with appropriate luminance contributions from each source. The mixture of the two wavelengths in appropriate quantities determines what stimulus chromaticity is produced. In this case, the appropriate quantities of monochromatic light to be mixed are expressed in terms of luminance of each wavelength. The calculation of the amounts of luminance required at each wavelength is discussed in a later section.

The graphical method described above is not only tedious, but also does not yield exact answers. To circumvent this, a mathematical solution may be implemented on a digital computer.

Trigonometry is used to compute a table of angles, theta, formed when drawing a line from the anchor wavelength to every point on the spectrum locus at one-nanometer intervals. The chromaticity coordinates of points on the spectrum locus are provided at one-nanometer intervals in texts such as Wyszecki and Stiles, 1982, pp. 725-735. Figure 3 illustrates this trigonometric solution. Figure 4 shows graphically the determination of theta (referenced to an anchor wavelength of 650 nm) for a specific example - 490 nm.

A copy of a FORTRAN program written to determine theta angles is found in Appendix A. From this FORTRAN program, once the anchor wavelength has been selected and its chromaticity coordinates entered, a table of thetas is compiled for each one-nanometer point around the

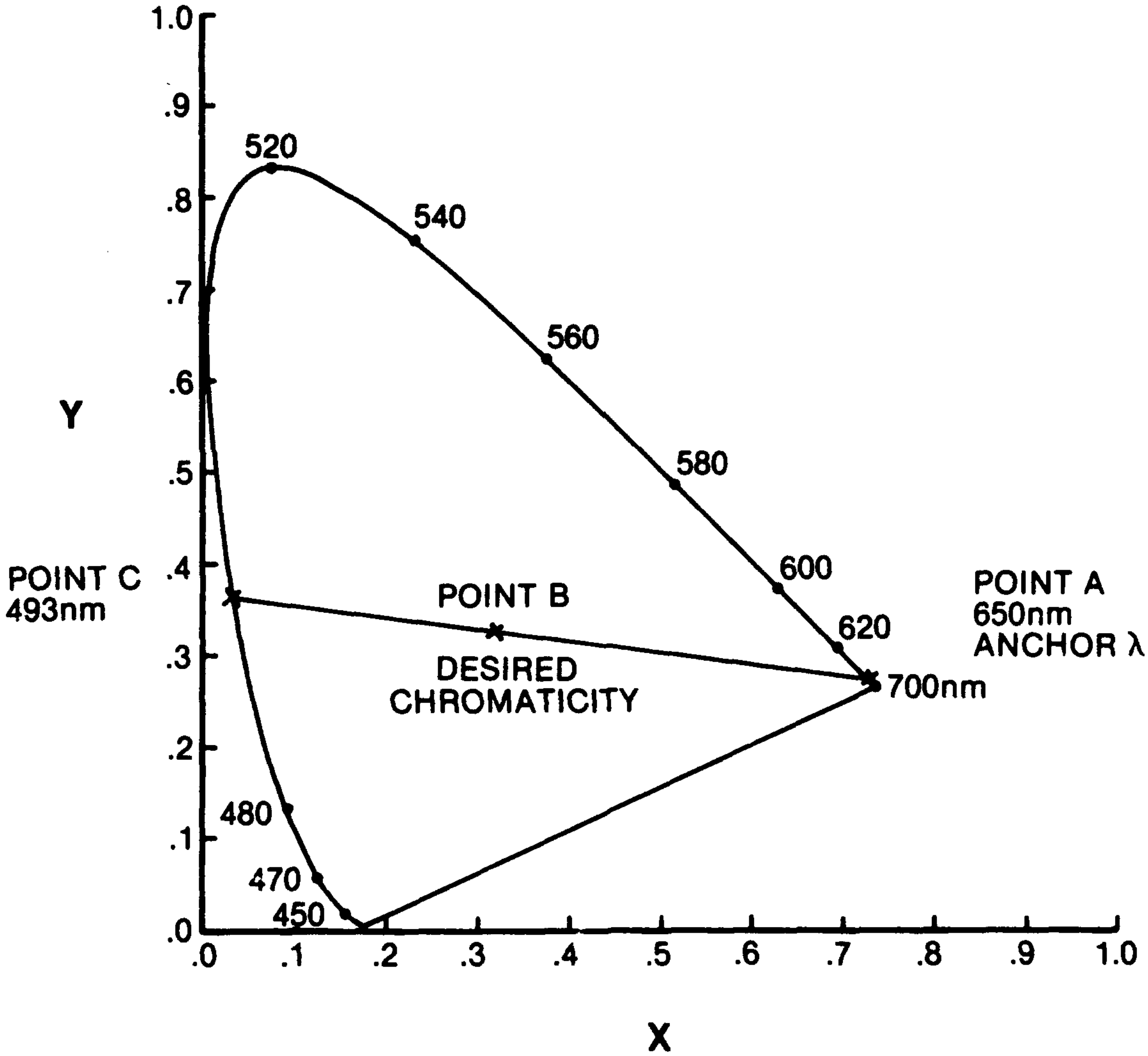

Figure 2. Trichromator Operation in 1931 CIE Chromaticity Space.

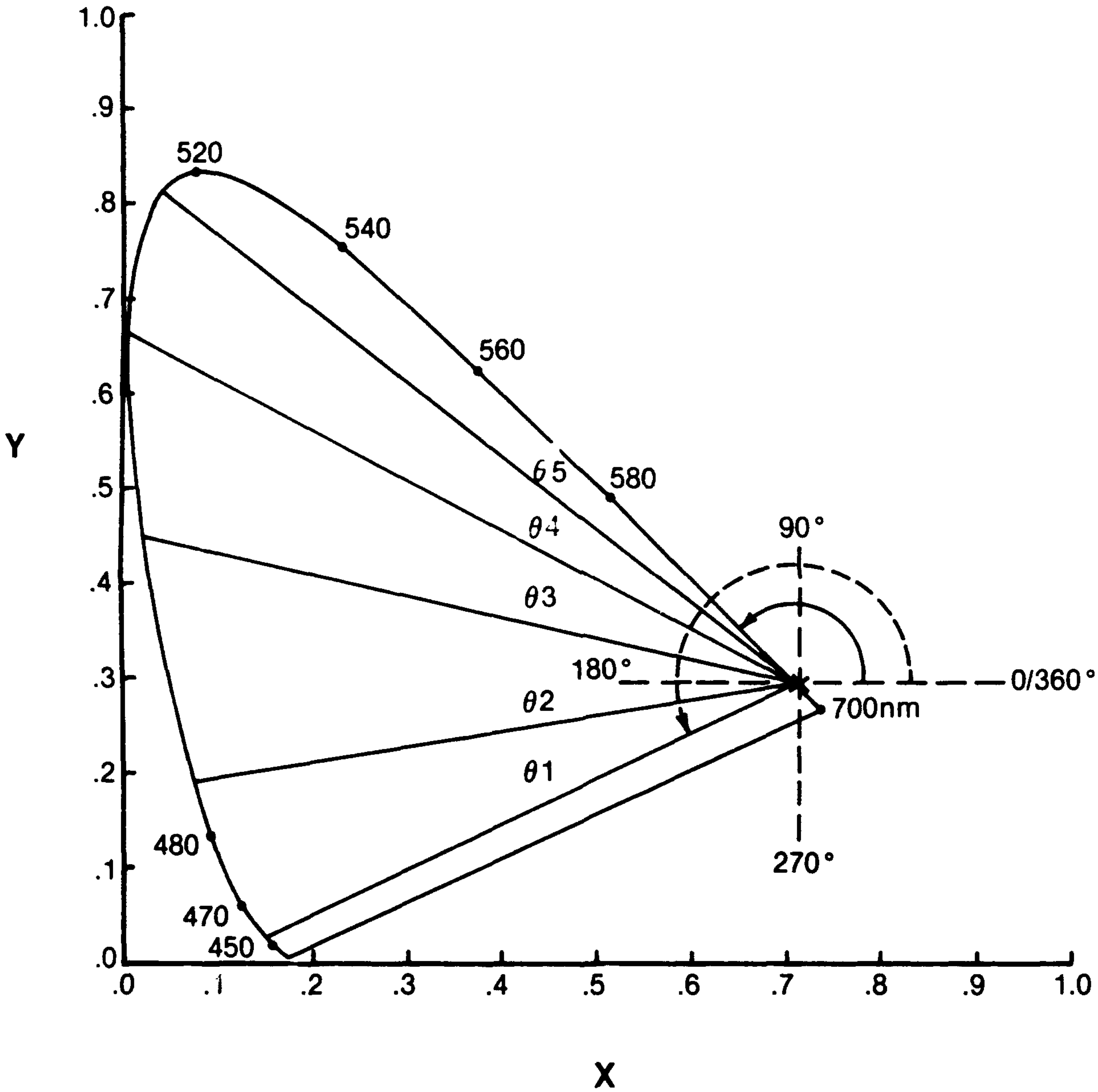

Figure 3. New Axis Coordinate Grid for Determination of Theta Angle.

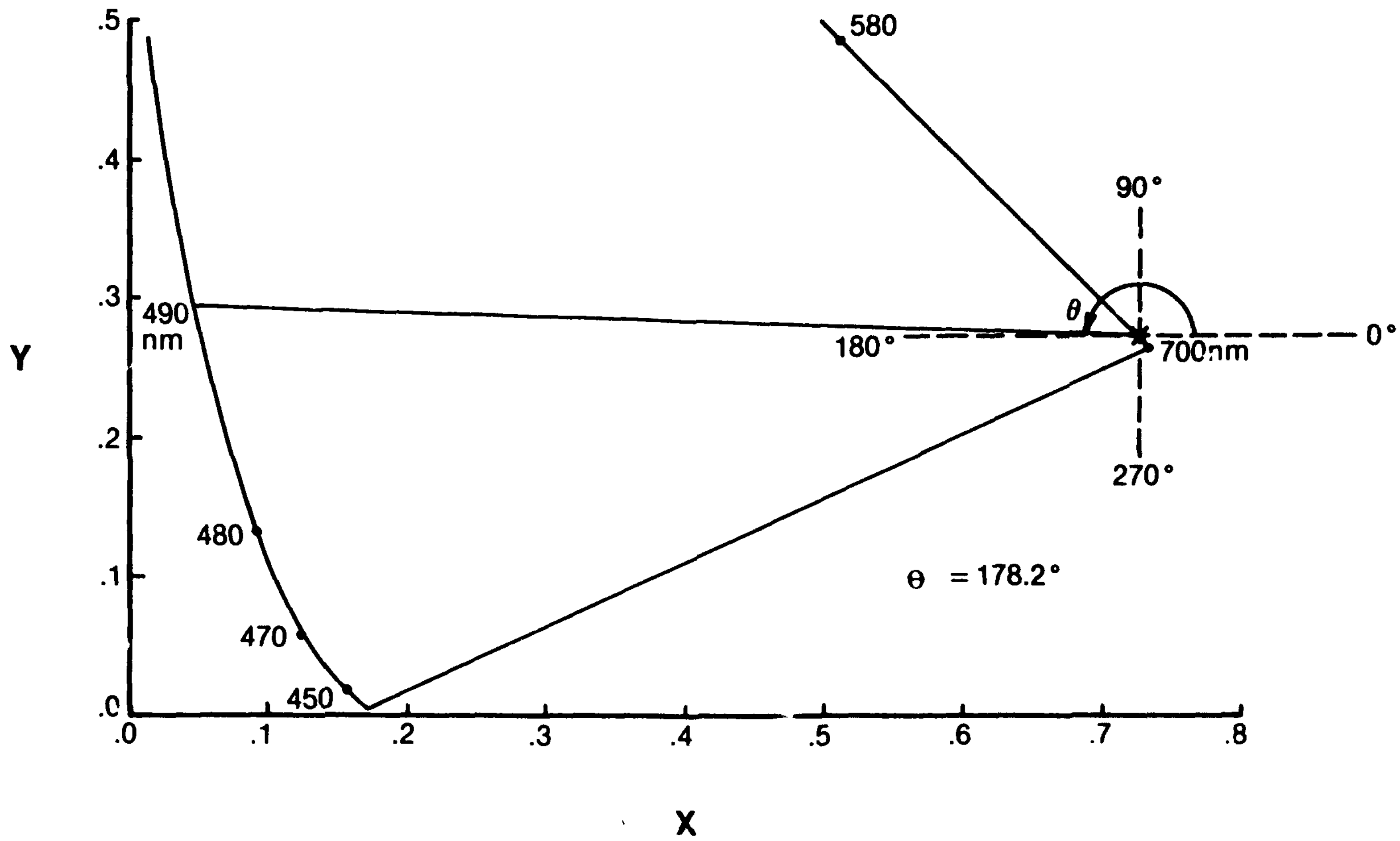

Figure 4. Example of use of Trigonometric Solution to Theta Angle.

spectrum locus (360-830 nm), referenced to the anchor wavelength. A compilation of thetas for an anchor wavelength of 650 nm is shown in Appendix B.

For the sake of convenience, this tabulated set of thetas is stored on a computer as a data file. A second computer program then can be written to search through the table of thetas to find the closest integer wavelengths, and then perform the necessary linear interpolation to arrive at the exact wavelength needed for mixing. A copy of a BASIC program written to achieve this purpose is found in Appendix C.

This section has summarized a mathematical solution to the problem of determining the wavelength required for mixing with an anchor wavelength to achieve a desired stimulus chromaticity. The next section describes a procedure for determining the luminances which are required to produce the desired chromaticity at a specific luminance.

4.2 Luminance Calculations

With the trichromator system, the luminance contributions from the three monochromators is controlled by a series of calibrated neutral density wedges. The author's practice has been to store this luminance calibration data on a computer as a data file.

The typical situation the researcher generally faces is one of trying to produce a specified chromaticity at a controlled luminance level. Once the anchor wavelength is chosen, two more steps remain. The first is to determine the variable dominant wavelength to produce the desired point on the chromaticity diagram. This step was outlined above in Section 4.1. The second step is to calculate the luminances required for the two dominant wavelengths to arrive at the needed luminance level.

In order to accomplish these two steps, the following equations are provided and were derived from Hunt (1975).

Given that:

$$X3 = \frac{\left[\left(\frac{X1}{Y1}\right) \times L1\right] + \left[\left(\frac{X2}{Y2}\right) \times L2\right]}{\left(\frac{L1}{Y1}\right) + \left(\frac{L2}{Y2}\right)} \tag{1}$$

$$Y3 = \frac{(L2 + L2)}{\left(\frac{L1}{Y1}\right) + \left(\frac{L2}{Y2}\right)} \tag{2}$$

and that $L1 + L2 = L3$ (3)

where:

$(X1,Y1)$ = CIE chromaticity coordinates for monochromator 1 wavelength setting (channel 1),

$(X2,Y2)$ = CIE chromaticity coordinates for the anchor wavelength setting (channel 2),

$(X3,Y3)$ = CIE chromaticity coordinates for the desired point inside chromaticity space,

$L1$ = luminance from channel 1,

$L2$ = luminance from channel 2,

and $L3$ = luminance of point of desired chromaticity.

Algebraic manipulation of the above equations results in the equations for the luminance necessary from each monochromator. The results of the algebraic manipulations are as follows:

$$L1 = ((-d / (c-d)) \times L3 \tag{4}$$

and

$$L2 = ((c / (c-d)) \times L3) \tag{5}$$

where:

$$c = ((Y3-Y1) / Y1)$$

and

$$d = ((Y3-Y2) / Y2)$$

5. EXAMPLE

It may be helpful to see a computational example using the above derived formulae. For this example, it is desired to produce a stimulus whose chromaticity coordinates are $x = 0.2$, $y = 0.7$ at a luminance of 500 cd/m^2.

The anchor wavelength for this example is 650 nm ($x = 0.72599$, $y = 0.27401$). The first task is to draw a straight line from the 650 nm point on the spectrum locus of the 1931 CIE diagram, through the desired chromaticity ($x = 0.2$, $y = 0.7$). Figure 5 illustrates this step.

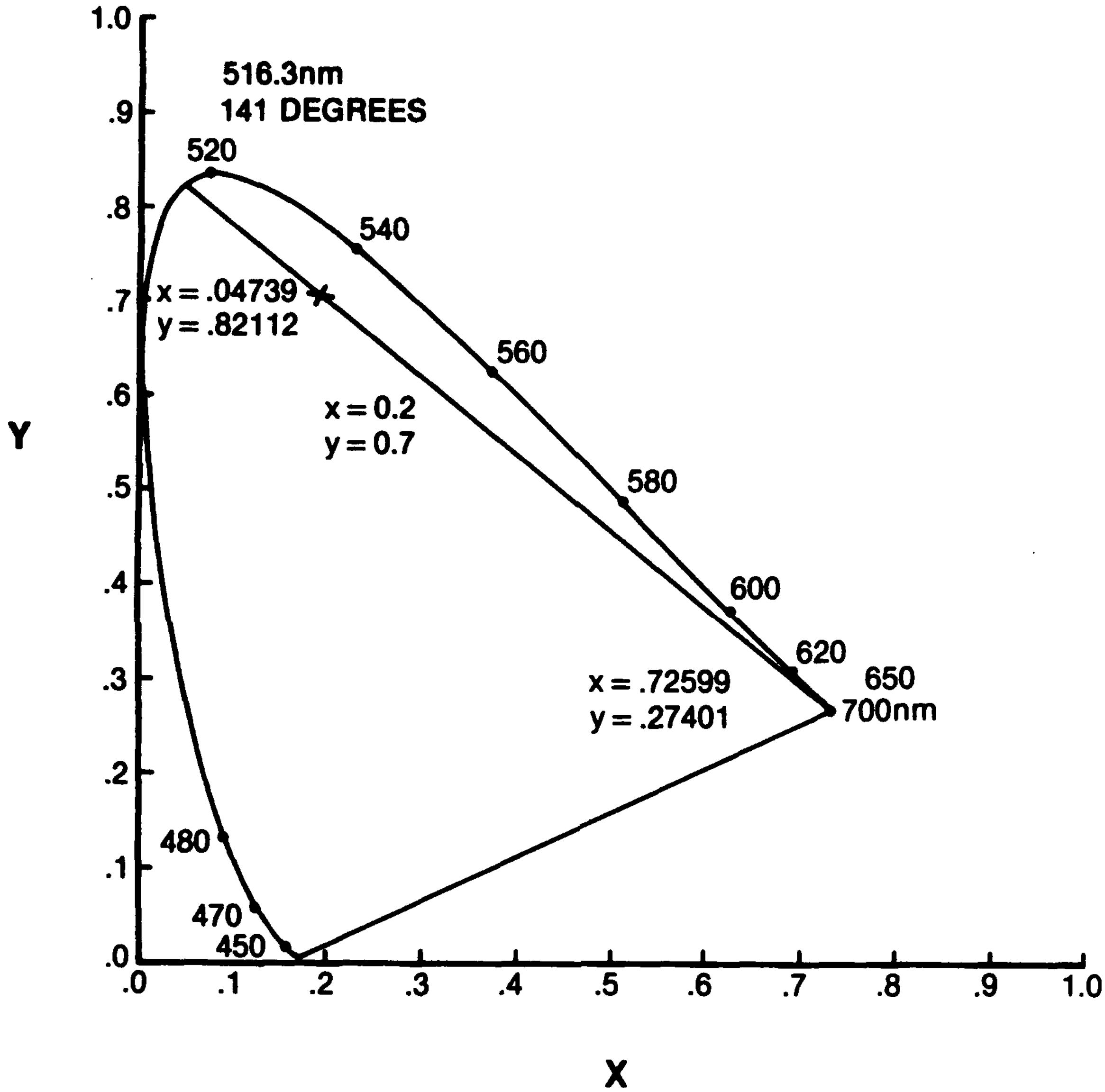

Figure 5. Finding Theta Angle and Spectrum Locus Intersection for Numerical Example.

The next task is to calculate the resulting theta angle. To avoid the ruler-and-protractor approach, the aforementioned computer programs were used to determine the theta angle of 140.997 degrees. Examining the data table of theta angles provided in Appendix B reveals the required monochromator setting needed for mixture with 650 nm is between 516 and 517 nanometers (between 141.3 and 140.7 degrees). A simple linear interpolation is performed to find the monochromator setting needed for mixture with the anchor wavelength. This calculation produces a wavelength of 516.3 nm.

Equations 4 and 5 define the luminance requirements from each of the two dominant wavelengths in order to produce the required stimulus at a given luminance level. In this example, the desired chromaticity of $x = 0.2$, $y = 0.7$ is to have a luminance of 500 cd/m^2.

Upon examining these equations, it can be seen that the variables needed for input are the (x,y) chromaticity coordinates for the following: anchor wavelength, variable wavelength setting for mixture with the anchor wavelength, and desired chromaticity within the 1931 CIE diagram. At this point, two of the three pairs of chromaticity coordinates are known - the anchor wavelength and the chromaticity of the desired point within the chromaticity diagram.

A linear interpolation must be performed on the x and y coordinates (between the chromaticity coordinates for 516 and 517 nm) to obtain the final pair of coordinates for the equations. Linear interpolation produces the resultant coordinates: $x = 0.04739$, $y = 0.82112$.

To use Equations 4 and 5, we define our variables as follows: let X1, Y1 be the coordinates of the variable monochromator in channel 1, X1 = 0.04739, Y1 = 0.82112, (516.3 nm in this example). Let X2, Y2 be the coordinates of the anchor wavelength, X2 = 0.72599, Y2 = 0.27401, (650 nm in this example). Let X3, Y3 be the coordinates of the desired chromaticity, X3 = 0.2, Y3 = 0.7.

Substituting values into Equation 4 we have:

$$L1 = \frac{\dfrac{-(0.7 - 0.27401)}{0.27401}}{\dfrac{0.7 - 0.82112}{0.82112} - \dfrac{0.7 - 0.27401}{0.27401}} \times 500 \qquad (6)$$

Solving Equation 6, we find that L1, the luminance contribution at 516.3 nm, is 456.8 cd/m^2. Thus, via Equation 3, the luminance required at 650 nm (L2) is 43.2 cd/m^2.

6. CONCLUSION

The numerical example worked in Section 5 is for producing one stimulus of a required chromaticity at a desired luminance level. The identical procedure would be followed to determine the other dominant wavelength needed to mix with the anchor wavelength to produce a second stimulus. In the same fashion as above, the luminance contributions would also be determined.

The theory of color mixing using a tri-stimulus colorimeter to produce two stimuli of known chromaticity and desired luminance has been presented. The derivation of the required formulae involve a simple algebraic manipulation of a set of equations. Since this manipulation has not heretofore been published in the literature, it is hoped that the sharing of this basic information will prove useful.

Appendix A - FORTRAN Program to Compute Theta Angles

```
C          FILENAME ANCHOR.FTN
C
C          GIVEN AN "ANCHOR" POINT, DRAW A CHORD TO EACH OF 471 POINTS ON
C          1931 CIE CHROMATICITY DIAGRAM, MEASURING ANGLES ACCORDING TO
C          THE FOLLOWING CONVENTION:
C                                             90
C
                                         180  ++  0
C
C                                            270
C
C
           DIMENSION X(471), Y(471), ANGLE(471)
           DATA Q/57.2958/
C
C          INPUT 471 PAIRS OF (X,Y) COORDINATES DEFINING CHROMATICITY
C          LOCUS FROM DISK FILE
C
           OPEN (UNIT=1,NAME='CMF31.DAT',TYPE='OLD',ACCESS='SEQUENTIAL')
           READ(1,*) (X(I), Y(I), I = 1, 471)
           CLOSE (UNIT=1)
C
C          INPUT X,Y COORDINATES OF ANCHOR POINT
C
           TYPE 10
      10   FORMAT(/,' X,Y COORDINATES OF ANCHOR POINT? ',$)
           ACCEPT *, XA, YA
C
C          COMPUTE ANGLES
C
           DO 100 I = 1, 471
C
C               IF Y(I) =YA , COMPUTE THETA DIFFERENTLY
C
                IF ( Y(I) .EQ. YA )  GO TO 20
C
C               FOUR POSSIBLE CASES:   XA >= XI, YA > YI
C                                      XA >= XI, YA < YI
C                                      XA <  XI, YA < YI
C                                      XA <  XI, YA > YI
C
C               COMPUTE THETA AND ADJUST IT ACCORDINGLY
C
                THETA = ATAN2 (XA - X(I), YA - Y(I)) * Q
C
                IF ( YA .LT. YI                    ) THETA = THETA + 180.
                IF ( XA .LT. XI .AND. YA .GT. YI ) THETA = THETA + 360.
                GO TO 60
```

```
C
   20         IF ( X(I) - XA )  30, 40, 50
C
   30         THETA = 90.
              GO TO 60
C
C             (ANCHOR POINT AND LOCUS POINT ARE IDENTICAL:  SET THETA = 0)
C
   40         THETA = 0.
              GO TO 100
C
   50         THETA = 270.
C
C             PERFORM FINAL MAPPING OF THETA
C
   60         THETA = 270. - THETA
              IF ( THETA .LT.   0. ) THETA = THETA + 360.
              IF ( THETA .GT. 360. ) THETA = THETA - 360.
C
  100         ANGLE(I) = THETA
C
C         SAVE RESULTS IN FILE
C
          OPEN (UNIT=1, NAME='LTHETA.DAT', TYPE='NEW', ACCESS='SEQUENTIAL')
          DO 105 I = 1, 471
  105         WRITE (1,110) I+359, ANGLE(I), X(I), Y(I)
  110         FORMAT (1X, I5, 3X, F5.1, 2(3X, F7.5))
          CLOSE(UNIT=1)
          STOP
          END
```

ANCHOR POINT = (.72599,.27401)

Appendix B - Wavelength by theta angle, referenced to 650 nm.

LAMBDA	THETA	X	Y	LAMBDA	THETA	X	Y	LAMBDA	THETA	X	Y	LAMBDA	THETA	X	Y
360	206.0	.17556	.00529	415	205.9	.17209	.00483	470	199.8	.12412	.05780	525	137.9	.11416	.82621
361	206.0	.17548	.00529	416	205.9	.17198	.00486	471	199.3	.12147	.06259	526	137.7	.12235	.82277
362	206.0	.17540	.00528	417	205.9	.17187	.00489	472	198.8	.11870	.06783	527	137.6	.13005	.81893
363	206.0	.17532	.00527	418	205.9	.17174	.00494	473	198.2	.11581	.07358	528	137.4	.13870	.81478
364	206.0	.17524	.00526	419	205.9	.17159	.00501	474	197.6	.11278	.07989	529	137.2	.14677	.81040
365	206.0	.17516	.00526	420	205.9	.17141	.00510	475	196.9	.10960	.08684	530	137.0	.15472	.80586
366	206.0	.17509	.00525	421	205.9	.17121	.00521	476	196.2	.10626	.09449	531	136.9	.16253	.80124
367	206.0	.17501	.00524	422	205.8	.17099	.00533	477	195.4	.10278	.10286	532	136.8	.17024	.79652
368	206.0	.17494	.00523	423	205.8	.17077	.00547	478	194.5	.09913	.11201	533	136.6	.17785	.79169
369	206.0	.17488	.00522	424	205.8	.17054	.00562	479	193.6	.09531	.12194	534	136.5	.18539	.78673
370	206.0	.17482	.00522	425	205.8	.17030	.00579	480	192.6	.09129	.13270	535	136.4	.19288	.78163
371	206.0	.17477	.00523	426	205.7	.17005	.00597	481	191.5	.08708	.14432	536	136.3	.20031	.77640
372	206.0	.17472	.00524	427	205.7	.16978	.00618	482	190.3	.08268	.15687	537	136.2	.20769	.77105
373	206.0	.17466	.00524	428	205.7	.16950	.00640	483	189.1	.07812	.17042	538	136.1	.21503	.76559
374	206.0	.17459	.00522	429	205.7	.16920	.00664	484	187.8	.07344	.18503	539	136.0	.22234	.76002
375	206.0	.17451	.00518	430	205.6	.16888	.00690	485	186.4	.06871	.20072	540	135.9	.22962	.75433
376	206.0	.17441	.00513	431	205.6	.16853	.00718	486	184.9	.06399	.21747	541	135.9	.23689	.74852
377	206.0	.17431	.00507	432	205.5	.16815	.00749	487	183.3	.05932	.23525	542	135.8	.24413	.74262
378	206.0	.17422	.00502	433	205.5	.16775	.00782	488	181.7	.05467	.25409	543	135.7	.25136	.73661
379	206.0	.17416	.00498	434	205.4	.16733	.00817	489	180.0	.05003	.27400	544	135.7	.25858	.73051
380	206.0	.17411	.00496	435	205.4	.16690	.00855	490	178.2	.04539	.29498	545	135.6	.26578	.72432
381	206.0	.17409	.00496	436	205.3	.16645	.00896	491	176.4	.04076	.31698	546	135.6	.27296	.71806
382	206.0	.17407	.00497	437	205.3	.16598	.00940	492	174.5	.03620	.33990	547	135.5	.28013	.71172
383	206.0	.17406	.00498	438	205.2	.16548	.00987	493	172.6	.03176	.36360	548	135.5	.28729	.70532
384	206.0	.17404	.00498	439	205.2	.16496	.01035	494	170.7	.02749	.38792	549	135.4	.29445	.69884
385	206.0	.17401	.00498	440	205.1	.16441	.01086	495	168.8	.02346	.41270	550	135.4	.30160	.69231
386	206.0	.17397	.00497	441	205.0	.16383	.01138	496	166.9	.01970	.43776	551	135.4	.30876	.68571
387	206.0	.17393	.00494	442	205.0	.16321	.01194	497	165.1	.01627	.46295	552	135.4	.31592	.67906
388	206.0	.17389	.00493	443	204.9	.16255	.01252	498	163.3	.01318	.48821	553	135.3	.32306	.67237
389	206.0	.17384	.00492	444	204.8	.16185	.01314	499	161.5	.01048	.51340	554	135.3	.33021	.66563
390	206.0	.17380	.00492	445	204.7	.16111	.01379	500	159.8	.00817	.53842	555	135.3	.33736	.65885
391	206.0	.17376	.00492	446	204.6	.16031	.01449	501	158.1	.00628	.56307	556	135.3	.34451	.65203
392	206.0	.17370	.00494	447	204.6	.15947	.01523	502	156.5	.00487	.58712	557	135.2	.35167	.64573
393	206.0	.17366	.00494	448	204.4	.15857	.01602	503	155.0	.00398	.61045	558	135.2	.35881	.63829
394	206.0	.17361	.00494	449	204.3	.15763	.01684	504	153.6	.00364	.63301	559	135.2	.36596	.63138
395	206.0	.17356	.00492	450	204.2	.15664	.01771	505	152.2	.00386	.65482	560	135.2	.37310	.62445
396	206.0	.17351	.00490	451	204.1	.15560	.01861	506	150.9	.00464	.67590	561	135.2	.38024	.61750
397	206.0	.17347	.00486	452	204.0	.15452	.01956	507	149.6	.00601	.69612	562	135.2	.38738	.61054
398	206.0	.17342	.00484	453	203.9	.15340	.02055	508	148.4	.00799	.71534	563	135.2	.39451	.60357
399	206.0	.17338	.00481	454	203.7	.15222	.02161	509	147.3	.01060	.73341	564	135.2	.40163	.59659
400	206.0	.17334	.00480	455	203.6	.15099	.02274	510	146.2	.01387	.75019	565	135.2	.40873	.58961
401	206.0	.17329	.00479	456	203.5	.14969	.02395	511	145.2	.01777	.76561	566	135.1	.41583	.58262
402	206.0	.17324	.00478	457	203.3	.14834	.02525	512	144.3	.02224	.77963	567	135.1	.42292	.57563
403	206.0	.17317	.00478	458	203.1	.14693	.02663	513	143.4	.02727	.79211	568	135.1	.42999	.56865
404	206.0	.17310	.00477	459	203.0	.14547	.02812	514	142.7	.03282	.80293	569	135.1	.43704	.56167
405	206.0	.17302	.00478	460	202.8	.14396	.02970	515	141.9	.03885	.81202	570	135.1	.44406	.55472
406	206.0	.17293	.00478	461	202.6	.14241	.03139	516	141.3	.04533	.81939	571	135.1	.45106	.54777
407	206.0	.17284	.00479	462	202.4	.14080	.03321	517	140.7	.05218	.82516	572	135.1	.45804	.54084
408	205.9	.17275	.00480	463	202.1	.13912	.03520	518	140.2	.05932	.82943	573	135.1	.46499	.53393
409	205.9	.17266	.00480	464	201.9	.13737	.03740	519	139.7	.06672	.83227	574	135.1	.47190	.52705
410	205.9	.17258	.00480	465	201.6	.13550	.03988	520	139.3	.07430	.83380	575	135.1	.47878	.52020
411	205.9	.17249	.00480	466	201.3	.13351	.04269	521	139.0	.08205	.83409	576	135.1	.48561	.51339
412	205.9	.17239	.00480	467	201.0	.13137	.04588	522	138.7	.08994	.83329	577	135.1	.49241	.50661
413	205.9	.17230	.00480	468	200.6	.12909	.04945	523	138.4	.09794	.83159	578	135.1	.49915	.49989
414	205.9	.17219	.00482	469	200.2	.12666	.05343	524	138.2	.10602	.82918	579	135.1	.50585	.49321

LAMBDA	THETA	X	Y	LAMBDA	THETA	X	Y	LAMBDA	THETA	X	Y	LAMBDA	THETA	X	Y
580	135.1	.51249	.48659	635	135.1	.71403	.28593	690	315.0	.73439	.26561	745	315.0	.73469	.26531
581	135.1	.51907	.48003	636	135.1	.71512	.28484	691	315.0	.73444	.26556	746	315.0	.73469	.26531
582	135.1	.52540	.47353	637	135.1	.71616	.28380	692	315.0	.73448	.26552	747	315.0	.73469	.26531
583	135.1	.53207	.46709	638	135.1	.71716	.28281	693	315.0	.73452	.26548	748	315.0	.73469	.26531
584	135.1	.53846	.46073	639	135.1	.71812	.28185	694	315.0	.73456	.26544	749	315.0	.73469	.26531
585	135.1	.54479	.45443	640	135.1	.71903	.28094	695	315.0	.73459	.26541	750	315.0	.73469	.26531
586	135.1	.55103	.44823	641	135.1	.71991	.28006	696	315.0	.73462	.26538	751	315.0	.73469	.26531
587	135.1	.55719	.44210	642	135.2	.72075	.27922	697	315.0	.73465	.26535	752	315.0	.73469	.26531
588	135.1	.56327	.43606	643	135.2	.72155	.27842	698	315.0	.73467	.26533	753	315.0	.73469	.26531
589	135.1	.56926	.43010	644	135.2	.72232	.27766	699	315.0	.73469	.26531	754	315.0	.73469	.26531
590	135.1	.57515	.42423	645	135.2	.72303	.27695	700	315.0	.73469	.26531	755	315.0	.73469	.26531
591	135.1	.58094	.41846	646	135.3	.72370	.27628	701	315.0	.73469	.26531	756	315.0	.73469	.26531
592	135.1	.58665	.41276	647	135.2	.72433	.27566	702	315.0	.73469	.26531	757	315.0	.73469	.26531
593	135.1	.59222	.40719	648	135.3	.72491	.27508	703	315.0	.73469	.26531	758	315.0	.73469	.26531
594	135.1	.59766	.40176	649	135.0	.72547	.27453	704	315.0	.73469	.26531	759	315.0	.73469	.26531
595	135.1	.60293	.39650	650	0.0	.72599	.27401	705	315.0	.73469	.26531	760	315.0	.73469	.26531
596	135.1	.60803	.39141	651	315.0	.72649	.27351	706	315.0	.73469	.26531	761	315.0	.73469	.26531
597	135.1	.61298	.38648	652	315.0	.72698	.27302	707	315.0	.73469	.26531	762	315.0	.73469	.26531
598	135.1	.61778	.38171	653	315.0	.72743	.27257	708	315.0	.73469	.26531	763	315.0	.73469	.26531
599	135.1	.62246	.37705	654	315.0	.72786	.27214	709	315.0	.73469	.26531	764	315.0	.73469	.26531
600	135.1	.62704	.37249	655	315.0	.72827	.27173	710	315.0	.73469	.26531	765	315.0	.73469	.26531
601	135.1	.63152	.36803	656	315.0	.72866	.27134	711	315.0	.73469	.26531	766	315.0	.73469	.26531
602	135.1	.63590	.36367	657	315.0	.72902	.27098	712	315.0	.73469	.26531	767	315.0	.73469	.26531
603	135.1	.64016	.35943	658	315.0	.72936	.27064	713	315.0	.73469	.26531	768	315.0	.73469	.26531
604	135.1	.64427	.35533	659	315.0	.72968	.27032	714	315.0	.73469	.26531	769	315.0	.73469	.26531
605	135.1	.64823	.35140	660	315.0	.72997	.27003	715	315.0	.73469	.26531	770	315.0	.73469	.26531
606	135.1	.65203	.34763	661	315.0	.73023	.26977	716	315.0	.73469	.26531	771	315.0	.73469	.26531
607	135.1	.65567	.34402	662	315.0	.73047	.26953	717	315.0	.73469	.26531	772	315.0	.73469	.26531
608	135.1	.65917	.34055	663	315.0	.73069	.26931	718	315.0	.73469	.26531	773	315.0	.73469	.26531
609	135.1	.66253	.33722	664	315.0	.73090	.26910	719	315.0	.73469	.26531	774	315.0	.73469	.26531
610	135.1	.66576	.33401	665	315.0	.73109	.26891	720	315.0	.73469	.26531	775	315.0	.73469	.26531
611	135.1	.66887	.33092	666	315.0	.73128	.26872	721	315.0	.73469	.26531	776	315.0	.73469	.26531
612	135.1	.67186	.32795	667	315.0	.73147	.26853	722	315.0	.73469	.26531	777	315.0	.73469	.26531
613	135.1	.67472	.32509	668	315.0	.73165	.26835	723	315.0	.73469	.26531	778	315.0	.73469	.26531
614	135.1	.67746	.32236	669	315.0	.73183	.26817	724	315.0	.73469	.26531	779	315.0	.73469	.26531
615	135.1	.68008	.31975	670	315.0	.73199	.26801	725	315.0	.73469	.26531	780	315.0	.73469	.26531
616	135.1	.68258	.31725	671	315.0	.73215	.26785	726	315.0	.73469	.26531	781	315.0	.73469	.26531
617	135.1	.68497	.31486	672	315.0	.73230	.26770	727	315.0	.73469	.26531	782	315.0	.73469	.26531
618	135.1	.68725	.31259	673	315.0	.73244	.26756	728	315.0	.73469	.26531	783	315.0	.73469	.26531
619	135.1	.68943	.31041	674	315.0	.73258	.26742	729	315.0	.73469	.26531	784	315.0	.73469	.26531
620	135.1	.69151	.30834	675	315.0	.73272	.26728	730	315.0	.73469	.26531	785	315.0	.73469	.26531
621	135.1	.69349	.30637	676	315.0	.73286	.26714	731	315.0	.73469	.26531	786	315.0	.73469	.26531
622	135.1	.69539	.30448	677	315.0	.73300	.26700	732	315.0	.73469	.26531	787	315.0	.73469	.26531
623	135.1	.69721	.30267	678	315.0	.73314	.26686	733	315.0	.73469	.26531	788	315.0	.73469	.26531
624	135.1	.69894	.30095	679	315.0	.73328	.26672	734	315.0	.73469	.26531	789	315.0	.73469	.26531
625	135.1	.70061	.29930	680	315.0	.73342	.26658	735	315.0	.73469	.26531	790	315.0	.73469	.26531
626	135.1	.70219	.29773	681	315.0	.73355	.26645	736	315.0	.73469	.26531	791	315.0	.73469	.26531
627	135.1	.70371	.29622	682	315.0	.73368	.26632	737	315.0	.73469	.26531	792	315.0	.73469	.26531
628	135.1	.70516	.29477	683	315.0	.73381	.26619	738	315.0	.73469	.26531	793	315.0	.73469	.26531
629	135.1	.70656	.29338	684	315.0	.73394	.26606	739	315.0	.73469	.26531	794	315.0	.73469	.26531
630	135.1	.70792	.29203	685	315.0	.73405	.26595	740	315.0	.73469	.26531	795	315.0	.73469	.26531
631	135.1	.70923	.29072	686	315.0	.73414	.26586	741	315.0	.73469	.26531	796	315.0	.73469	.26531
632	135.1	.71050	.28945	687	315.0	.73422	.26578	742	315.0	.73469	.26531	797	315.0	.73469	.26531
633	135.1	.71173	.28823	688	315.0	.73429	.26571	743	315.0	.73469	.26531	798	315.0	.73469	.26531
634	135.1	.71290	.28706	689	315.0	.73434	.26566	744	315.0	.73469	.26531	799	315.0	.73469	.26531

LAMBDA	THETA	X	Y	LAMBDA	THETA	X	Y	LAMBDA	THETA	X	Y	LAMBDA	THETA	X	Y
800	315.0	.73469	.26531												
801	315.0	.73469	.26531												
802	315.0	.73469	.26531												
803	315.0	.73469	.26531												
804	315.0	.73469	.26531												
805	315.0	.73469	.26531												
806	315.0	.73469	.26531												
807	315.0	.73469	.26531												
808	315.0	.73469	.26531												
809	315.0	.73469	.26531												
810	315.0	.73469	.26531												
811	315.0	.73469	.26531												
812	315.0	.73469	.26531												
813	315.0	.73469	.26531												
814	315.0	.73469	.26531												
815	315.0	.73469	.26531												
816	315.0	.73469	.26531												
817	315.0	.73469	.26531												
818	315.0	.73469	.26531												
819	315.0	.73469	.26531												
820	315.0	.73469	.26531												
821	315.0	.73469	.26531												
822	315.0	.73469	.26531												
823	315.0	.73469	.26531												
824	315.0	.73469	.26531												
825	315.0	.73469	.26531												
826	315.0	.73469	.26531												
827	315.0	.73469	.26531												
828	315.0	.73469	.26531												
829	315.0	.73469	.26531												
830	315.0	.73469	.26531												

Appendix C - BASIC Program to Calculate Wavelength Needed for Mixing with Anchor Wavelength to Produce Desired Pair of Chromaticity Coordinates.

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10 REM   THIS PROGRAM CALCULATES WAVELENGTH NEEDED FOR MIXING, ITS X,Y
15 REM   COORDINATES ON THE SPECTRUM
20 REM LOCUS AND X,Y COORDINATES FOR THE DESIRED CHROMATICITY
40 REM ACHROMATIC POINT IS SET AT D65
60 REM IF YOU WANT TO CHANGE IT, ALTER THE U',V' COORDINATES IN LINE 80
80 LET X0=.1978 LET\Y0=.4683
90 LET Q=57.2958
100 PRINT "ENTER X COORDINATE OF REQUIRED DOMINANT WAVELENGTH;"\INPUT X1
110 PRINT "ENTER Y COORDINATE OF REQUIRED DOMINANT WAVELENGTH;"\INPUT Y1
120 PRINT "ENTER THE FILE NAME YOU WANT THIS DATA STORED IN "\INPUT I$
130 REM CONVERTS 1931 CIE to 1976 UCS
140 LET X=4*X1/(-2*X1+12*Y1+3)
150 LET Y=9*Y1/(-2*X1+12*Y1+3)
160 IF (X-X0)<>0GO TO 200
170 A=90
180 IF (Y-Y0)>0GO TO 280
190 A=270\GO TO 280
200 L=(X-X0)/(Y-Y0)
210 A=ATN(L)*Q
220 IF A > 0GO TO 250
230 IF X>X0GO TO 270
240 A=90-A\GO TO 280
250 IF (X-X0)>0GO TO 275
260 IF (Y-Y0)>0GO TO 280
270 A=270-A\GO TO 280
275 LET A=90-A
300 REM X9, Y9 ARE X, Y COORDINATES FOR TEST FIELD
310 PRINT "WHAT IS X COORDINATE OF DESIRED COLOR?"\INPUT X9
320 PRINT "WHAT IS Y COORDINATE OF DESIRED COLOR?"\INPUT Y9
380 REM SETS MONOCHROMATOR 2 AT ANCHOR OF 655NM
390 LET X2=.72827/LET Y2=.27173
410 REM CALCULATES LENGTH OF R2 FROM 655 NM(MONOCHROMATOR) TO TEST POINT
420 IF (X9-X2)<>0GO TO 460
430 A2=90
440 IF (Y9-Y2)>0GO TO 550
450 A2=270\GO TO 550
460 L9=(X9-X2)/(Y9-Y2)
470 A2=ATN(L9)*Q
480 REM A2 IS THETA FOR DESIRED CHROMATICITY POINT
490 IF A2 > 0GO TO 520
500 IF X9 > X2GO TO 540
510 A2=90-A2\GO TO 550
520 IF (X9-X2) > 0GO TO 545
530 IF (Y9-Y2)>0GO TO 550
```

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540 A2=278-A2\GO TO 550
545 LET A2=90-A2
550 OPEN "DXO:LTHETA.DAT" FOR INPUT AS FILE VF1
560 OPEN I$ FOR OUTPUT AS FILE VF2
570 FOR N=1 TO 451
580 LET T2=VF1((4*N)-2)
590 IF A2>T2GO>TO 660
600 IF A2=T2GO TO 620
610 NEXT N
620 LET L4=VF1((4*N)-3)
630 LET X3=VF1((4*N)-1)
640 LET Y3-VF1(4*N)
650 GO TO 850
660 LET T1=VF1((4*N)-6)
670 REM FINDS ONE THETA ABOVE T2 AND STORES IT AS T1
680 LET L2=VF1((4*N)-3)
690 REM L2 IS WAVELENGTH ASSOCIATED WITH THETA
700 LET L1=VF1((4*N)-7)
710 REM L1 IS WAVELENGTH ASSOCIATED WITH THETA ABOVE
720 REM LINEAR INTERPOLATION FOLLOWS FOR LAMBDA AND X AND Y COORDS
730 L3=(T2-A2)*(L2-L1)/(T2-T1)
740 REM L3 IS AMOUNT TO CHANGE L2 BY TO RECEIVE REQUIRED LAMBDA
750 L4=(L2-L3)
760 REM L4 IS LAMBDA ASSOCIATED WITH WAVELENGTH NEEDED FOR MIXING WITH ANCHOR
770 LET X8=VF1((4*N)-1)
780 LET Y8-VF1(4*N)
790 LET X7=VF1((4*N)-5)
800 LET Y7=VF1((4*N)-4)
810 LET X4=(T1-A2)*(X7-X8)/(T1-T2)
820 LET X3=(X7-X4)
830 LET Y4=(T1-A2)*(Y7-Y8)/(T1-T2)
840 LET Y3=(Y7-Y4)
850 PRINT "MONOCHROMATOR WAVELENGTH FOR MIXING IS"L4
880 PRINT "X,Y FOR MONOCHROMATOR ARE "X3,Y3
882 PRINT "X,Y FOR DESIRED CHROMATICITY ARE"X9,Y9
885 PRINT "U',V' COORDINATES FOR DESIRED CHROMATICITY ARE"X6,Y6
890 LET VF2(1)=X3
900 LET VF2(2)=Y3
910 LET VF2(3)=L4
920 LET VF2(4)=X9
930 LET VF2(5)=Y9
940 END
```

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